HW 6.6b

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Name:______________________________ Date:__________________

Math 9 Honours Section 6.6b Solving Equations with Trigonometry

1. Evaluate each expression without a calculator:

a) $\cos 90° + 5\sin 270°$ $0 + 5(-1)$ $= -5$	b) $5\sin 180° + 4\cos 0°$ 5 $= 4$	c) $\sin 90° - 3\cos 180°$ $1 - 3(-1)$ $= 1 + 3$ $= 4$
d) $6\cos(-270°) + \sin(-90°)$ $= -1$	e) $3\sin 45° - 4\cos 150°$ $3\left(\frac{\sqrt{2}}{2}\right) - 4\left(-\frac{\sqrt{3}}{2}\right)$ $\frac{3\sqrt{2} + 4\sqrt{3}}{2}$	f) $-2\sin(225°) + \frac{2}{3}\tan(135°)$ $-2\left(-\frac{1}{\sqrt{2}}\right) + \frac{2}{3}(-1)$ $\frac{2\sqrt{2}}{2} - \frac{2}{3}$
g) $2\tan^2 120° + 3\sin^2 60°$ $= \frac{33}{4}$	h) $-3\cos^2 150° - 3\sin^2(-225°)$ $= -3$	i) $-3\sin^2 300° - 3\cos^2(-60°)$ $= \frac{3}{2}$

2. Solve for θ between $0° \le \theta \le 360°$

a) $3\sin\theta - 3 = 0$ $3\sin\theta = 3$ $\sin\theta = 1$ $\theta = \sin^{-1}(1)$ $\theta = 90°$	b) $\sqrt{2}\cos\theta + 1 = 0$ $\cos\theta = -\frac{1}{\sqrt{2}}$ $\theta = 135°$, $\theta = 225°$	c) $\sqrt{2}\sin\theta - 1 = 0$ $\sin\theta = \frac{1}{\sqrt{2}}$ $\theta = 45°$ $\theta = 135°$
d) $2\cos\theta + \sqrt{3} = 0$ $2\cos\theta = -\sqrt{3}$ $\cos\theta = -\frac{\sqrt{3}}{2}$ $\theta_1 = 150°$ $\theta_2 = 210°$	e) $\tan\theta - \sqrt{3} = 0$	f) $2\tan\theta + 2\sqrt{3} = 0$

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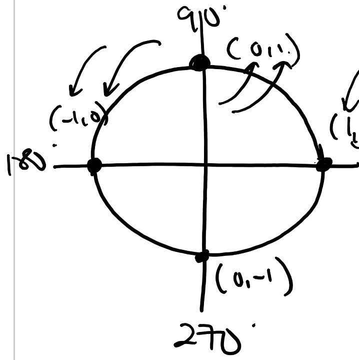
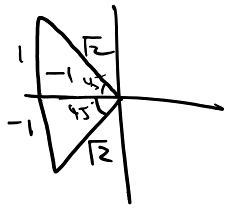


$\tan\theta = \frac{\sin\theta}{\cos\theta}$



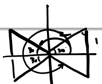
g) $\sin^2\theta - 1 = 0$

$(\sin\theta)^2 = 1$

$\sin\theta = 1$ or $\sin\theta = -1$

$\theta = 90^\circ$ $\quad$ $\theta = 270^\circ$

h) $4\sin^2\theta - 1 = 0$



$4\sin^2\theta = 1$

$(\sin^2\theta) = \frac{1}{4}$

$\sin\theta = +\frac{1}{2}$ or $\sin\theta = -\frac{1}{2}$

$\theta = 30^\circ, 150^\circ$ $\quad$ $\theta_2 = 210^\circ, 330^\circ$

i) $4\cos^2\theta - 3 = 0$

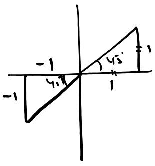
3. Solve for θ between $0^\circ \le \theta \le 360^\circ$

a) $\frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta = 1$

$\theta_1 = 45^\circ, 225^\circ$



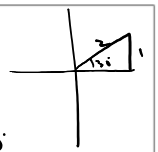
b) $2\sin^2\theta = \sin\theta$

$2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0$ $\quad$ $\sin\theta = \frac{1}{2}$

$\theta_1 = 0^\circ, 180^\circ, 360^\circ$ $\quad$ $\theta = 30^\circ, 150^\circ$



c) $2\sin^2\theta = \sin\theta + 1$

$2A^2 = A + 1$

$2A^2 - A - 1 = 0$

$(2A + 1)(A - 1)$

$A = -\frac{1}{2}$ $\quad$ $A = 1$

$\boxed{\sin\theta = -\frac{1}{2}}$ $\quad$ $\sin\theta = 1$

$\sin\theta = -\frac{1}{2}$ $\quad$ $\boxed{\theta = 210^\circ, 330^\circ}$

$\sin\theta = 1$ $\quad$ $\boxed{\theta = 90^\circ}$

d) $2\sin^2\theta = \sin\theta + 5$

$2A^2 - A - 5 = 0$

(quadratic formula)

$\sin\theta = \frac{1 \pm \sqrt{1+40}}{4}$

$= \frac{1 \pm \sqrt{41}}{4}$ $\quad$ $\varnothing$

e) $2\cos^2\theta + 7\cos\theta = 4$

$2A^2 + 7A - 4 = 0$

$(2A - 1)(A + 4) = 0$

$\cos\theta = \frac{1}{2}, -4$

$\theta = 60^\circ, 300^\circ$

f) $2\sin^2\theta - 11\sin\theta = 6$

$2A^2 - 11A - 6 = 0$

$(2A + 1)(A - 6) = 0$

$\sin\theta = -\frac{1}{2}, 6$ (6 doesn't work)

$\theta = 210^\circ, 330^\circ$

g) $(4\sin^2 - 1)(\sin^2\theta - 1) = 0$

$(2A+1)(2A-1)(A+1)(A-1) = 0$

$\sin\theta = -\frac{1}{2}, \frac{1}{2}, -1, 1$

$\theta = 210^\circ, 330^\circ, 30^\circ, 150^\circ, 270^\circ, 90^\circ$

h) $\cos^2\theta - 3\sin\theta + 1 = 0$

$\cos^2\theta = 1 - \sin^2\theta$

$(1 - A^2) - 3A + 1 = 0$

$-A^2 - 3A + 2 = 0$

$A^2 + 3A - 2 = 0$

$\sin\theta = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 + \sqrt{17}}{2}$ ($\frac{-3-\sqrt{17}}{2}$ is invalid)

$\sin\theta \doteq 34.1633^\circ, 145.8367^\circ$

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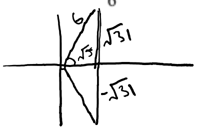
$(\sin^2\theta + \cos^2\theta = 1)$

i) $2\sin\theta \times \cos\theta = \sin\theta$ $2\sin\theta \cdot \cos\theta - \sin\theta = 0$ $\sin\theta(2\cos\theta - 1) = 0$ $\sin\theta = 0$ or $\cos\theta = \frac{1}{2}$ $\theta = 0°, 180°, 360°, 60°, 300°$	j) $7 + 4\cos\theta - 4\sin^2\theta = 0$ $\sin^2\theta = 1 - \cos^2\theta$ $-4(1 - A^2) + 4A + 7 = 0$ $-4 + 4A^2 + 4A + 7 = 0$ $4A^2 + 4A + 3 = 0$ $\varnothing$
k) $(2\sin^3\theta - 2\sin^2\theta) - (\sin\theta + 1) = 0$ $2\sin^2\theta(\sin\theta - 1) - (\sin\theta - 1) = 0$ $(2\sin^2\theta - 1)(\sin\theta - 1) = 0$ $\downarrow$ $\varnothing$ $\quad$ $\downarrow$ $\sin\theta = 1$ $\theta = 90°$	$4\cos^4\theta + 3\cos^2\theta = 1$ $4A^4 + 3A^2 - 1 = 0$ $(4A^2 - 1)(A^2 + 1) = 0$ $(2A + 1)(2A - 1)(A^2 + 1) = 0$ $\downarrow -\frac{1}{2}$ $\quad$ $\downarrow \frac{1}{2}$ $\quad$ $\downarrow \varnothing$ $\cos\theta = \pm\frac{1}{2}$ $\theta = 60°, 120°, 240°, 300°$

4. If $\cos\theta = \frac{\sqrt{5}}{6}$, then what is the exact value of $\sin\theta$ and $\tan\theta$

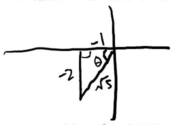


opposite side $= \sqrt{6^2 - 5} = \sqrt{36 - 5} = \sqrt{31}$

$\sin\theta = \pm\frac{\sqrt{31}}{6}$

$\tan\theta = \pm\sqrt{\frac{31}{5}}$

5. If $\tan\theta = 2$ and θ is in quadrant III, then what is the exact value of $\sin\theta$ *and* $\cos\theta$?



hypotenuse $= \sqrt{1^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$

$\sin\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{2}$

$\cos\theta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

6. Given $0° \le \theta \le 360°$, if $\sin\theta = k$ and there is only one solution, what are the possible value(s) of k?

1, -1

7. Given $0° \le \theta \le 360°$, if $\sin\theta = k$ and there are three solutions, what are the possible value(s) of k?

0

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8. Suppose $A + B = 180^\circ$, then which of the following statements are true?

i) $\sin A = \sin B$ ii) $\cos A = \cos B$ iii) $\tan A = -\tan B$

9. If $0^\circ \le \theta \le 360^\circ$ then what is the minimum value and maximum value of the expression:

$2\sin^2\theta + \cos^2\theta + 1$

$= \sin^2\theta + \sin^2\theta + \cos^2\theta + 1$
$= (\sin^2\theta + \cos^2\theta) + \sin^2\theta + 1$
$= 1 + \sin^2\theta + 1$
$= \sin^2\theta + 2$

$-1 \le \sin\theta \le 1$
$(-1)^2 = 1^2 = 1$
if $\sin\theta = \pm 1$
$(\pm 1)^2 + 2 = 3$
(max.)

if $\sin\theta = 0$
$0^2 + 2 = 2$
(min)

10. How many solutions will the following equation have? $\sin\theta \times \cos\theta \times \tan\theta = 0$

if $\sin\theta = 0$
$\theta = 0^\circ, 180^\circ, 360^\circ$
if $\cos\theta = 0$
$\theta = 90^\circ, 270^\circ$
if $\tan\theta = 0$
$\theta = 0^\circ, 180^\circ, 360^\circ$

$\theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$
5 solutions

11. Given that $x\cos\theta + y\sin\theta = 4$ and $x\sin\theta - y\cos\theta = 3$, then which of the following statements is correct?

i) $x + y = 5$ ii) $x + y = 7$ iii) $x^2 + y^2 = 5$ iv) $x^2 + y^2 = 25$

$(x\cos\theta + y\sin\theta)^2 = 4^2$
$(x\sin\theta - y\cos\theta)^2 = 3^2$
$\begin{cases} x^2\cos^2\theta + 2xy(\sin\theta)(\cos\theta) + y^2\sin^2\theta = 16 \\ x^2\sin^2\theta - 2xy(\sin\theta)(\cos\theta) + y^2\cos^2\theta = 9 \end{cases}$
$x^2(\cos^2\theta + \sin^2\theta) + y^2(\sin^2\theta + \cos^2\theta) = 25$

$(1)x^2 + (1)y^2 = 25$
$x^2 + y^2 = 25$

12. If $0^\circ \le \theta \le 2016^\circ$, how many angles satisfy the equation: $\sin^2 2016^\circ + \sin^2\theta = 1$ (CNML 2016).

$\sin^2 2016 = \sin^2 216$
$\sin^2 216 + \sin^2\theta = 1$
$\sin^2\theta = 1 - \sin^2 216$
$\sin^2\theta \doteq 0.654508497$
$\sin\theta \doteq \pm 0.809017$
$\theta = 54^\circ, 126^\circ, 234^\circ, 306^\circ$

$360^\circ \times 5 = 1800^\circ$
Each angle goes around 5 times
$2016^\circ - 1800^\circ = 216^\circ$
$54^\circ < 216^\circ$
$126^\circ < 216^\circ$
54° & 126° goes around an extra once
$4 \times 5 + 2 = 22$ angles

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